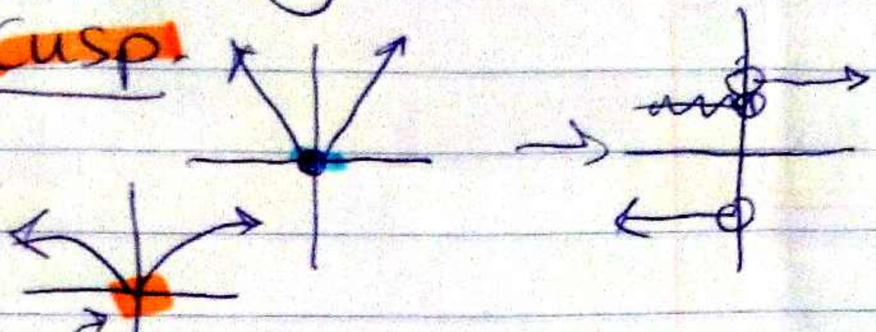
Differentiability

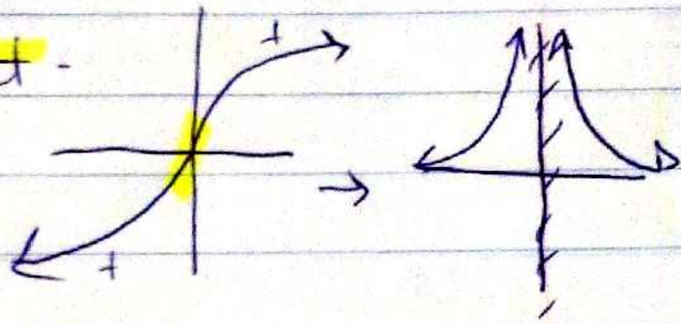
① Corner / Cusp.

pointy spot



cusp (creates an asymptote)

② Vertical Tangent.



③ Discontinuity - Hole, Asymptote, Jump.

ex) Find all points where $f(x) = |x+2| + 3$ is not differentiable

$x = -2$

Theorem

Differentiability Implies Continuity

If f has a derivative at $x = a$ (is differentiable) then f is continuous at ~~x~~ $= a$

not other way around.

The Power Rule

If $f(x) = ax^n$, $f'(x) = (n \cdot a)x^{n-1}$.

** The derivative of a constant is always 0.

ex) $f(x) = x^6 - 3x^3 + 4x^2 + \frac{6}{x} + 2$

$= x^6 - 3x^3 + 4x^2 + 6x^{-1} + 2$

$f'(x) = 6x^5 - 9x^2 + 8x - 6x^{-2}$

$= 6x^5 - 9x^2 + 8x - \frac{6}{x^2}$

① Simplify
② Power Rule
③ Simplify

ex) $f(x) = \frac{3x^4 + 2x^2}{x} = 3x^3 + 2x$

$f'(x) = 9x^2 + 2$

ex) $f(x) = \frac{3x^4 - 2x^3 + 5}{4x^2}$ Find the tangent line at $(1, 3/2)$

① $f(x) = \frac{3}{4}x^2 - \frac{1}{2}x + \frac{5}{4}x^{-2}$

$f'(x) = \frac{3}{2}x - \frac{1}{2} - \frac{5}{2}x^{-3}$

② $f'(1) = 3/2(1) - 1/2 - 5/2(1)^{-3} = -3/2 = m$

③ $y = \frac{-3}{2}(x - 1) + \frac{3}{2} = \frac{-3}{2}x + 3$

$$y = \frac{s}{s^{1/2} - 1}$$

$u = s \qquad v = s^{1/2} - 1$

$u' = 1 \qquad v' = \frac{1}{2} s^{-1/2}$

$$y' = \frac{s^{1/2} - 1 - \frac{1}{2} s^{-1/2} \cdot s}{(s^{1/2} - 1)^2}$$

$s^{1/2}$

$$= \frac{\frac{1}{2} s^{1/2} - 1}{(s^{1/2} - 1)^2}$$

$$y = \frac{\cos t}{t^3}$$

$u = \cos t \qquad v = t^3$

$u' = -\sin t \qquad v' = 3t^2$

$$y' = \frac{-t^3 \sin t - 3t^2 \cos t}{t^6}$$

$$= \frac{-t^2 (t \sin t + 3 \cos t)}{t^6}$$

$$= \frac{-(t \sin t + 3 \cos t)}{t^4}$$

$$y = \frac{3 - 2x - x^2}{x^2 - 1}$$

$u = 3 - 2x - x^2 \qquad v = x^2 - 1$

$u' = -2 - 2x \qquad v' = 2x$

$$y' = \frac{(-2 - 2x)(x^2 - 1) - 2x(3 - 2x - x^2)}{(x^2 - 1)^2}$$

$$= \frac{-2x^2 - 2x^3 + 2x + 2 - 6x + 4x^2 + 2x^3}{(x^2 - 1)^2}$$

$$= \frac{2x^2 - 4x + 2}{(x^2 - 1)^2}$$

$$y = x \sin x + \cos x$$

$u = x \qquad v = \sin x$

$u' = 1 \qquad v' = \cos x$

$$y' = \sin x + x \cos x - \sin x$$

$$= x \cos x$$